

Starting Advanced Mathematics

The Essential Foundation

Test exercise 1

1 Simplify, where possible, the following expressions.

- | | | |
|--------------------|-------------------|-----------------------|
| (a) $a+b+c$ | (b) $a+2b-b+2a$ | (c) $x-2z-y+2z$ |
| (d) $2lm+l-m$ | (e) $abc-b^2$ | (f) $a\times b+c$ |
| (g) $a+b\times c$ | (h) $2p\times p$ | (i) $q\times 3q^2$ |
| (j) $pq\times 2pq$ | (k) $4x\times 4x$ | (l) $2xy\times 4xy^2$ |

2 If $x=2$, $y=-3$ and $z=4$ find the values of the following expressions.

- | | | |
|-------------------|---------------------|-----------------------|
| (a) $x+y$ | (b) $x-2z$ | (c) $z-x+y$ |
| (d) $z-x-y$ | (e) $2x-3y-4z$ | (f) x^2+z^2 |
| (g) y^2+z^2 | (h) $2xy$ | (i) $2xy-yz$ |
| (j) $3z\times 3z$ | (k) $2yz\times 8yz$ | (l) $2xy\times 4xy^2$ |

3 A rectangular shaped room is to have wallpaper on the walls and ceiling. The length, breadth and height of the room are x metres, y metres and z metres respectively. The area A m² to be papered, neglecting doors and windows, is given by

$$A = xy + 2xz + 2yz.$$

If $x=4.6$, $y=3.2$ and $z=2.8$, calculate the area to be papered, giving your answer correct to the nearest square metre. (You will need a calculator.)

Test exercise 2

1 Expand the brackets in each of the following expressions to get a single number.

- | | | |
|---------------|---------------|----------------|
| (a) $3(2-5)$ | (b) $2(4+1)$ | (c) $5+(1-2)$ |
| (d) $6-(1+3)$ | (e) $6-(1-3)$ | (f) $8-2(3-6)$ |

2 Expand the brackets in the following expressions and simplify, if possible.

- | | |
|----------------------|-------------------------------|
| (a) $a(a+b)+b(a+b)$ | (b) $a(a-b)-b(a+b)$ |
| (c) $z(z-3)+3(z-3)$ | (d) $z(z-3(z+3))-z(z+3(z-3))$ |
| (e) $(2x)^2+(-2x)^2$ | (f) $(2x)^3+(-2x)^3$ |

Test exercise 3

1 Solve the following equations, checking that your answers are correct.

(a) $2 - x = 3$

(b) $4 + 2z = -10$

(c) $1 - 3z = 15 + 4z$

(d) $2 = 4k - 3 + k$

(e) $-3u + 2 = 2 + u$

(f) $2v - 7 = 2 + v$

2 Find the solution of each of the following equations. In each case, check your answers.

(a) $2(w - 3) = 10$

(b) $5(x - 1) = 2(x + 2)$

(c) $4 - 2y = 3(1 - y)$

(d) $4c + 3(2 - 3c) = 16$

(e) $5(2 - t) = 2(t - 1) + 4(t + 3)$

(f) $2(h + 1) - 4(h - 1) = -2 + 3(h - 4)$

Test exercise 4

1 Carry out the following multiplications and divisions, giving your answers as fractions in their lowest terms.

(a) $\frac{2}{3} \times \frac{5}{4}$

(b) $\frac{5}{6} \times \frac{2}{3}$

(c) $\frac{3}{7} \times \frac{14}{15}$

(d) $\frac{4}{7} \div \frac{3}{14}$

(e) $\frac{8}{9} \div \frac{16}{27}$

(f) $\frac{18}{35} \div \frac{15}{28}$

(g) $2 \times \frac{3}{4}$

(h) $1 \div \frac{3}{4}$

2 Carry out the following calculations, giving your answers as fractions in their lowest terms.

(a) $\frac{3}{5} + \frac{1}{2}$

(b) $\frac{3}{4} - \frac{4}{7}$

(c) $\frac{2}{3} + \frac{3}{4} + \frac{5}{6}$

(d) $\frac{2}{3} + \frac{3}{4} - \frac{5}{6}$

(e) $\frac{2}{5} + \frac{3}{4} - \frac{3}{10}$

(f) $\frac{8}{7} - \frac{3}{4} - \frac{1}{3}$

(g) $\frac{1}{2} + \frac{1}{4} - \frac{1}{8}$

(h) $\frac{1}{2} \times \frac{1}{4} - \frac{1}{8}$

Test exercise 5

1 Carry out the following multiplications and divisions, giving your answers as fractions in their lowest terms.

(a) $\frac{5}{6} \times \frac{2}{3}$

(b) $\frac{2}{xy} \times \frac{x}{y^2}$

(c) $\frac{3}{y} \times \frac{2}{x}$

(d) $\frac{x^2}{3} \times \frac{6}{x}$

(e) $\frac{8}{9} \div \frac{16}{27}$

(f) $\frac{x}{2} \div \frac{x^2}{4}$

(g) $\frac{2}{5}a^2 \div \frac{3}{5}a$

(h) $\frac{12x}{7y} \div \frac{3x^2}{14y^2}$

2 Carry out the following calculations, giving your answers as fractions in their lowest terms.

(a) $\frac{5}{6} + \frac{2}{3}$

(b) $\frac{a}{2} + \frac{b}{4}$

(c) $\frac{1}{3x} + \frac{5}{6x}$

(d) $\frac{3p}{q} - \frac{2p}{3q}$

(e) $\frac{1}{n} + \frac{n}{2}$

(f) $\frac{3p}{q} + \frac{q}{p}$

(g) $4 - \frac{5}{x}$

(h) $\frac{3x}{2} - \frac{1}{5x}$

3 Carry out the following calculations, giving your answers as fractions in their lowest terms.

(a) $\frac{1}{2} + \frac{1}{6} - \frac{1}{4}$

(b) $\frac{x}{4} - \frac{x}{2} + \frac{x}{3}$

(c) $\frac{1}{n} + \frac{2}{n} + \frac{3}{n}$

(d) $\frac{p}{2} \times \frac{q}{4}$

(e) $\frac{3}{4}a \div \frac{2}{3}a$

(f) $\frac{1}{2} \times \frac{1}{5} + \frac{1}{3}$

(g) $\frac{x}{4} \times \frac{6}{x^2} - \frac{1}{x}$

(h) $\frac{a}{3} + \frac{b}{2} \times \frac{3a}{b}$

Test exercise 6

1 Simplify the following.

(a) $(2^2 \times 3 \times 5^2) \times (2 \times 3^2 \times 5^3)$

(b) $(2^2 \times 3 \times 5^2) \div (2 \times 3^2 \times 5^3)$

(c) $\frac{2x}{3y} \times \frac{6y}{z^2} \times \frac{z^3}{2x^3}$

(d) $\left(\frac{2a}{3b}\right)^2 \times \left(\frac{3b}{c}\right)^3$

(e) $\left(\frac{3r^2}{2s}\right)^3 \div \left(\frac{3s}{4t}\right)^2$

(f) $\left(\frac{2l}{3m}\right)^3 \times \left(\frac{mn}{2l}\right)^2 \div \left(\frac{6lm}{n^3}\right)^2$

Test exercise 7

1 Solve the following equations, checking that your answers are correct.

(a) $2 - x = 3$

(b) $4 + 2z = -10$

(c) $1 - 3z = 15 + 4z$

(d) $2 = 4k - 3 + k$

(e) $-3u + 2 = 2 + u$

(f) $2v - 7 = 2 + v$

2 Find the solution of each of the following equations. In each case, check your answers.

(a) $2(w - 3) = 10$

(b) $5(x - 1) = 2(x + 2)$

(c) $4 - 2y = 3(1 - y)$

(d) $4c + 3(2 - 3c) = 16$

(e) $5(2 - t) = 2(t - 1) + 4(t + 3)$

(f) $2(h + 1) - 4(h - 1) = -2 + 3(h - 4)$

3 Find the solution of each of the following equations.

(a) $\frac{1 - 2x}{3} - \frac{1 + 3x}{4} = 10$

(b) $\frac{x}{6} - \frac{x - 2}{3} = 4$

(c) $\frac{2}{3}(x + 1) - \frac{3}{5}(2x - 3) = \frac{1}{3}$

(d) $\frac{4}{7}(2x - 1) - \frac{1}{2}(x - 5) = 2$

(e) $\frac{1 + 2x}{5} - \frac{5 + 3x}{4} = \frac{x - 4}{2} - 5$

(f) $\frac{2}{3}(x + 4) - \frac{3}{4}(2x + 5) = \frac{1}{12}(x - 2)$

Test exercise 8

1 Factorise the following expressions.

(a) $ax + 2x$

(b) $pq + p^2$

(c) $2p^2 + 3q + 6 + pq$

(d) $4z - 3t + 12 - tz$

(e) $2h^2 + 3h - 2hk - 3k$

(f) $ax + ay - az$

2 Expand the following brackets, simplifying your answer where possible.

(a) $(a + 2)(c - 2)$

(b) $(2p + 1)(p - 1)$

(c) $(x - y)(x - 2z)$

(d) $(x + 3y)(x - 3y)$

(e) $(y - 3z)^2$

(f) $(2p + 3q)^2$

Test exercise 9

1 Solve each of the following equations for x .

(a) $ax + bx = x(a - b) + c$ (b) $s = \frac{1}{2}n(x + y)$ (c) $b(x + a) = ab$

(d) $x(b + c) - a = b$ (e) $\frac{x}{a} + \frac{y}{b} = \frac{xy}{ab}$ (f) $\frac{x^3}{a^3} + \frac{y^3}{b^3} = 1$

2 In each case, make the letter given at the end the subject of the formula.

(a) $s = \frac{a}{1 - r}$, r (b) $A = 2\pi r(r + h)$, h (c) $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$, f

(d) $h = \frac{1}{2}gt^2$, t (e) $v^2 = w^2(a^2 - x^2)$, x (f) $V = \frac{1}{3}\sqrt{\frac{s^3}{8\pi}}$, s

Test exercise 10

1 Factorise, where possible, each of the following quadratics.

(a) $3x^2 + 2x$ (b) $1 + 4x + 3x^2$ (c) $9 - 25y^2$
(d) $3p^2 + 4p - 4$ (e) $3x^2 - 15x + 12$ (f) $6s^2 + 54$
(g) $4x - 5x^2$ (h) $4a^2 - 20a + 25$ (i) $2 - 2w^2$
(j) $12x^2 + x - 6$ (k) $3 + 5x - 4x^2$ (l) $8t^2 + 16t - 10$
(m) $12 - 36x + 27x^2$ (n) $2 + 4x^2$ (o) $42 + 8x - 2x^2$

Test exercise 11

1 Find the roots, if any, of the following quadratic equations. Where appropriate give your answers correct to 3 significant figures.

(a) $2x^2 - 7x + 3 = 0$ (b) $2x^2 - 7x - 4 = 0$ (c) $2x^2 - 4x - 7 = 0$
(d) $3x^2 + 4x + 5 = 0$ (e) $4x^2 - 5x - 6 = 0$ (f) $x^2 + x - 1 = 0$

Test exercise 12

1 Solve the following pairs of simultaneous equations.

(a) $\begin{cases} x - y = 8 \\ x + y = 13 \end{cases}$ (b) $\begin{cases} x + 2y = 17 \\ 2x + 3y = 16 \end{cases}$ (c) $\begin{cases} 5x - 8y = 4 \\ 2x + 3y = -17 \end{cases}$
(d) $\begin{cases} 2x - 4y = 6 \\ 2x + 4y = 6 \end{cases}$ (e) $\begin{cases} 5x = 1 - 4y \\ 5y = 1 - 6x \end{cases}$ (f) $\begin{cases} 0.2x - \frac{1}{2}y = -1 \\ 0.1x + \frac{1}{5}y = 4 \end{cases}$

